

DPP-01

SINGLE CORRECT TYPE

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1. Let x_1 and x_2 be the roots of the quadratic equation $x^2 + px + q = 0$.

If $x_1 = \frac{x_2 + 4}{2x_2 - 1}$, then the value of $(2q + p)$ is equal to

- (A) 1 (B) 2 (C) 3 (D) 4

Ans. (D)

Sol. Given $2x_1x_2 - x_1 = x_2 + 4$

$$2x_1x_2 - (x_1 + x_2) = 4$$

$$2q + p = 4$$

2. The value of the expression

$x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = \cot \frac{\pi}{12}$, is

- (A) 2 (B) 1 (C) 0 (D) 3

Ans. (B)

Sol. We know $\cot \frac{\pi}{12} = 2 + \sqrt{3}$

$$\text{Let } f(x) = x^4 - 8x^3 + 18x^2 - 8x + 2$$

Substituting $x = 2 + \sqrt{3}$, we get

$$f(2 + \sqrt{3}) = (2 + \sqrt{3})^4 - 8(2 + \sqrt{3})^3 + 18(2 + \sqrt{3})^2 - 8(2 + \sqrt{3}) + 2$$

$$= (2 + \sqrt{3})^2((2 + \sqrt{3})^2 + 18) - 8(2 + \sqrt{3})((2 + \sqrt{3})^2 + 1) + 2$$

$$= (7 + 4\sqrt{3})(4\sqrt{3} + 25) - 8(2 + \sqrt{3})(8 + 4\sqrt{3}) + 2$$

$$= 175 + 48 - 128 - 96 + 2$$

$$= 1$$

3. If $P(x) = ax^2 + bx + c$ and

$Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x) \cdot Q(x) = 0$ has

- (A) exactly one real root (B) at least two real roots
(C) exactly three real roots (D) all four are real roots.

Ans. (B)

Sol. $P(x) = ax^2 + bx + c$

Hence, for real roots,

$$b^2 - 4ac \geq 0$$

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$$Q(x) = -ax^2 + dx + c$$

Hence, for real roots,

$$d^2 + 4ac \geq 0$$

Now, $ac \equiv 0$

Now if $ac < 0$

Then,

$$b^2 - 4ac > 0$$

Hence, $P(x) \cdot Q(x)$ has atleast 2 real roots.

If $ac > 0$

Then

$$d^2 + 4ac > 0$$

Hence, $P(x) \cdot Q(x)$ has atleast 2 real roots.

4. If α, β are the roots of the equation $x^2 + px + r = 0$ and $\alpha/3, 3\beta$ are the roots of the equation $x^2 + qx + r = 0$, then r equals

- (A) $\frac{3}{8}(p-3q)(3p+q)$ (B) $\frac{3}{8}(p+3q)(3p-q)$
(C) $\frac{3}{64}(3p-q)(p-3q)$ (D) $\frac{3}{64}(3p-q)(p-q)$

Ans. (C)

Sol. For quadratic equation $x^2 - px + r = 0$

product of roots = $\alpha\beta = r$ and sum of roots = $\alpha + \beta = p$.

For quadratic equation $x^2 - qx + r = 0$, product of roots = $\alpha\beta = r$ and sum of roots = $\frac{\alpha}{3} + 3\beta =$

$$q \Rightarrow \alpha + 9\beta = 3q..$$

Subtracting equation (1) from (2),

$$\alpha + 9\beta = 3q - \alpha - \beta = -p$$

$$8\beta = 3q - p \Rightarrow \beta = \frac{3q - p}{8}$$

Now putting the value of β in equation (1),

$$\alpha + \frac{3q - p}{8} = p$$

$$\Rightarrow \alpha = p - \frac{3q - p}{8} \Rightarrow \alpha = \frac{3(-3p + q)}{8}$$

$$\text{Thus, } r = \alpha\beta = \frac{3(3p - q)}{8} \times \frac{(3q - p)}{8} = \frac{3}{64}(3p - q)(q - 3p)$$

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5. Let λ_1 and λ_2 be two values of λ for which the expression $x^2 + (2 - \lambda)x + \lambda - \frac{3}{4}$ becomes a perfect square. The value of $(\lambda_1^2 + \lambda_2^2)$ equals

(A) 8 (B) 25 (C) 50 (D) 100

Ans. (C)

Sol. Given, $f(x) = x^2 + (2 - \lambda)x + \left(\lambda - \frac{3}{4}\right)$

Now, for a quadratic expression to become a perfect square, its $D = 0$

$$\Rightarrow (2 - \lambda)^2 = 4\left(\lambda - \frac{3}{4}\right) \Rightarrow 4 + \lambda^2 - 4\lambda = 4\lambda - 3$$

$$\Rightarrow \lambda^2 - 8\lambda + 7 = 0 \Rightarrow (\lambda - 7)(\lambda - 1) = 0$$

$$\therefore \lambda = 1, 7$$

$$\text{Hence, } (\lambda_1^2 + \lambda_2^2) = (1)^2 + (7)^2$$

$$= 1 + 49 = 50$$

6. If e^λ and $e^{-\lambda}$ are the roots of equation $3x^2 - (a + b)x + 2a = 0$, $a, b, \lambda \in \mathbb{R}$, $\lambda \neq 0$ then least integral value of b is

(A) 4 (B) 5 (C) 9 (D) 10

Ans. (B)

Sol. Sum of the roots in the given quadratic equation $= \frac{a+b}{3}$

$$\text{and the product of the roots} = \frac{2a}{3}$$

$$\text{Roots of the equation are } e^\lambda \text{ and } \frac{1}{e^\lambda}$$

$$\therefore e^\lambda \times \frac{1}{e^\lambda} = 1 = \frac{2a}{3} \Rightarrow a = \frac{3}{2}$$

$$e^\lambda + \frac{1}{e^\lambda} = \frac{a+b}{3} = \frac{2b+3}{6}$$

We know that the value of $e^\lambda + \frac{1}{e^\lambda} > 2$, when $\lambda \neq 0$

$$\therefore \frac{2b+3}{6} > 2 \Rightarrow b > 4.5$$

Therefore, the minimum integral value of $b = 5$

7. If α, β are the roots of the quadratic equation $x^2 + 2(1 - \cos 3\theta)x - 2\sin^2 3\theta = 0$ ($\theta \in \mathbb{R}$), then the maximum value of $\alpha^2 + \beta^2$ is equal to

(A) 0 (B) 4 (C) 8 (D) 16

Ans. (C)

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Sol. $x^2 + 2(1 - \cos 3\theta)x - 2\sin^2 3\theta = 0$

$$\alpha + B = -2(1 - \cos 3\theta)$$

$$\alpha^2 + B^2 = (\alpha + B)^2 - 2\alpha B.$$

$$\alpha B = -2\sin^2 3\theta$$

$$= (-2(1 - \cos 3\theta))^2 - 2(-2\sin^2 3\theta).$$

$$= 4(1 + \cos^2 3\theta - 2\cos 3\theta) + 4\sin^2 3\theta$$

$$= 4[1 + (\cos^2 3\theta + \sin^2 3\theta) - 2\cos 3\theta]$$

$$= 4(1 + 1 - 2\cos 3\theta)$$

$$= 8(1 - \cos 3\theta) \leq 8(1 + 1) = 16$$

$$\Rightarrow (\max \text{ of } \alpha^2 + B^2 = 16. (\text{when } \cos 3\theta = -1))$$

MULTIPLE CORRECTTYPE

8. If $(a + b + c) > 0$ and $a < 0 < b < c$, then the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ has

(A) real and distinct roots

(B) roots are imaginary

(C) product of roots is negative

(D) product of roots is positive

Ans. (D)

Sol. Given equation can be written as

$$(a + b + c)x^2 - (ab + bc + ca)x + 3 = 0$$

$$\therefore D = 4(ab + bc + ca)^2 - 12abc(a + b + c) > 0 \text{ as } abc < 0, a + b + c > 0$$

$\Rightarrow$ Roots of the equation are real.

9. If one of the roots of the equation $4x^2 - 15x + 4p = 0$ is the square of the other then the value of p is

(A) $125/64$

(B) $-27/8$

(C) $-125/8$

(D) $27/8$

Ans. (A)

Sol. $4x^2 - 15x + 4p = 0$

Let one root be a other root be a^2

$$\therefore a + a^2 = \frac{15}{4}$$

$$\therefore a^3 = \frac{4P}{4} = P$$

$$\therefore a = (P)^{1/3}$$

$$\therefore (P)^{1/3} + P(2/3) = \frac{15}{4}$$

Solving above we get

$$P = \frac{125}{64}$$

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MATCH THE COLUMN

10. Consider the quadratic equation $(k^2 - 1)x^2 + (2k^3 + 9k^2 + 3k - 14)x + (2k^3 + 5k^2 - 11k - 14) = 0$ then find the sum of all the value(s) of k (where $k \in \mathbb{R}$) for which the given equation has

Column-I

Column-II

- | | |
|--|--------------------|
| (A) Exactly one root zero and another root is finite | (P) -1 |
| (B) Both roots zero | (Q) $\frac{-5}{2}$ |
| (C) Exactly one root infinity | (R) 2 |
| (D) Both roots infinity | (S) $\frac{-7}{2}$ |
| | (T) 1 |

Ans. Q

10. $(k - 1)(k + 1)x^2 + (2k + 7)(k - 1)(k + 2)x + (2k + 7)(k - 2)(k + 1) = 0$
- (A) Exactly one root is '0' and other root is finite. P.O.R. = 0 but S.O.R. $\neq 0$ Therefore $k = 2$.
- (B) Both roots are zero means S.O.R. = P.O.R. = 0 Hence, $k = \frac{-7}{2}$ only
- (C) Exactly one root infinity i.e., coefficient of $x^2 = 0$ but coefficient of $x \neq 0$ Therefore $k = -1$ only.
- (D) Both roots are infinity i.e., coefficient of $x^2 =$ coefficient of $x = 0$ but constant term $\neq 0$
- Therefore $k = 1$